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ORIGINAL PAPER

Segmented taper equations with crown ratio and stand density for Dahurian Larch (*Larix gmelinii*) in Northeastern China

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Abstract: Segmented taper equation was selected to model stem profile of Dahurian larch (*Larix gmelinii* Rupr.). The data were based on stem analysis of 74 trees from Dailing Forest Bureau in Heilongjiang Province, Northeastern China. Two taper equations with crown ratio and stand basal area were derived from the Max and Burkhart's (1976) taper equation. Three taper equations were evaluated: (1) the original equation, (2) the original equation with crown ratio, and (3) the original equation with basal area. SAS NLIN and SYSNLIN procedures were used to fit taper equations. Fit statistics and cross-validation were used to evaluate the accuracy and precision of these models. Parameter estimates showed that the original equation with inclusion of crown ratio and basal area variables provided significantly different parameter estimates with lower standard errors. Overall fit statistics indicated that the root mean square error (RMSE) for diameter outside and inside bark decreased respectively by 10% and 7% in the original model with crown ratio and by 12% and 7.2% in the original model with basal area. Cross-validation further confirmed that the original equation with inclusion of crown ratio and basal area variables provided more accurate predictions at the lower section (relative heights, <10%) and upper section (relative heights, >50%) for both outside and inside bark diameters.

Keywords: basal area; crown ratio; *Larix gmelinii*; nonlinear regression; taper equations

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Introduction

Taper functions are the mathematical expression of the change in stem diameter as a function of stem height based on tree species, stand age, density, and many factors that affect site quality. In most cases, taper functions utilize the measurements of total height, diameter at breast height (DBH) and height above the ground as independent variables, since these variables are easily measured during common forest inventory activities (Brooks et al. 2008). Several taper functions of varying complexity have appeared in literatures over a century. All of these models can be classified into three major classes: simple taper functions (e.g. sharma and oderwald 2001; Zakrzewski and MacFarlane 2006), segmented taper functions (e.g. Max and Burkhart 1976; Fang et al. 2000; Jiang et al. 2005), variable form taper functions (e.g. Kozak 1988; Bi 2000). According to Kozak (2004), taper functions provide estimates of: (1) inside bark diameter at any point along the stem, (2) total stem volume, (3) merchantable height to any top diameter and from any stump height, and (4) individual log volumes of any length at any height above the ground.

Larson (1963) found that most variations in stem form are attributed to changes in the size of live crown and the length of the branch-free bole. Valenti and Cao (1986) replaced some coefficients of the Max and Burkhart's (1976) taper function with functions of crown ratio (CR) for loblolly pine (*Pinus taeda* L.) in Lousiana. Their results indicated that the inclusion of CR provided a significant improvement in description of loblolly pine taper. Muhairwe et al. (1994) found that incorporating crown ratio into the variable-form exponent of Kozak's (1988) taper function improved the fit of Kozak's taper function for lodgepole pine (*Pinus contorta* Dougl.). Jiang et al. (2007) evaluated the effectiveness of incorporating CR as an independent variable in their segmented polynomial taper function. They found that adding CR variable into taper equation significantly improved prediction of stem diameter and volume for yellow-poplar (*Liriodendron tulipifera* L.) in West Virginia. Furthermore, stem taper is affected by stand thinning (Sharma and Zhang 2004). There were significant taper differences between trees growing in

thinned and unthinned stands. Although other stand variables have been added to the taper equation studied in the past, little attempt has been made to quantify the stand density effect on tree taper.

Past studies incorporating crown ratio in taper equations generally divided the crown ratios into classes. Taper equation was fitted separately to each crown ratio class and the parameter estimates were regressed against the crown ratio classes to detect trends (Valenti and Cao 1986). This approach limits the sample size to the possible crown ratio classes and stand density. Recently mixed-effects modeling approach were used in taper equations (Leites and Robinson 2004; Trincado and Burkhart 2006). This approach captures more variation of tree taper among plots or trees. Random effects should be biologically related to tree and stand variables. The use of mixed-effects techniques allowed investigating the relationships between random parameters and crown ratio and standing density. Dahurian larch is the most widely planted commercial tree species in Northeastern China. Segmented taper equations have not been used to represent stem profiles in China. The objectives of this study were to address the relationship between crown ratio and stand density variables and parameter estimates in the segmented taper equation by Max and Burkhart (1976) for Dahurian larch and evaluate the effects of crown ratio and stand density on the stem taper.

Materials and methods

Data

Twenty four 0.04-ha plots (tree age ranging from 7 to 37 years old) were selected in Dahurian larch plantations, which are located at Dailing Forest Bureau in Heilongjiang Province, Northeastern China (128°37′E, 46°50′N). Ranges of these stand characteristics were the basal area of 2.2–37.1 m^2/ha (mean value, 20.43 ± 10.17 m^2), stand density of 846–4,050 trees/ha (2,005 ± 968 trees/ha), and dominant height of 5.4–23.3 m (14.36 ± 4.25 m). For each plot, three or four trees were randomly selected and destructively sampled. In total, there were 74 trees felled for stem analysis. Before felling, three attributes were measured for each sample tree: (1) diameter at breast height, dbh (1.3 m above ground); (2) total tree height, H; (3) height to the crown base, HCB, defined as the lowest whorl with three or more living branches. Crown ratio was calculated by subtracting HCB from total height and then dividing the result by total height. The trees were then felled, bucked into sections, and stem disks were removed at different heights above the tree base. Measurement intervals above breast height varied between 1 m and 2 m depending on the total tree height. Each disk was labeled and sealed in a plastic bag to preserve moisture and prevent shrinkage. In the laboratory, diameters outside and inside bark of each disk were measured along the largest axis and the smallest axis, and then arithmetically averaged. Descriptive statistics for the Dahurian larch sample trees are shown in Table 1.

Table 1. Summary statistics for Dahurian larch sample trees

	DBH (cm)	Total height (m)	Disk dob (cm)	Disk dib (cm)	Disk height (m)	Crown length (m)	Crown ratio
Mean	14.01	13.75	10.75	9.74	5.38	7.67	0.61
S.D.	5.71	5.44	6.3	5.9	4.89	2.33	0.19
Minimum	3.61	4.25	0.7	0.5	0	2.44	0.25
Maximum	25.52	23.7	34	30.35	21	13.8	0.97

Taper equation

The Max and Burkhart (1976) segmented polynomial taper equation was used in this study. This equation has been shown to provide accurate results for many species (Brooks et al. 2007; Brooks et al. 2008; Jiang and Brooks 2008). It consists of three equations that describe the neiloid of the lower section, the paraboloid frustum of the middle section, and the conical shape of the upper section. The three equations are grafted into one equation at two join points. This equation is of the form:

$$y = b_1(x-1) + b_2(x^2-1) + b_3(a_1-x)^2 I_1 + b_4(a_2-x)^2 \quad (1)$$

where $y = d^2/D^2$, d is the diameter outside or inside bark (cm) to measurement point at height h, D is the diameter outside bark at breast height (cm), $x = h/H$, h is the height above the ground to the measurement point (m), H is total tree height (m), a_i is the join points to be estimated from the sample data. $i = 1, 2$, b_i is regression coefficients, $i = 1 \ldots 4$,

$$I_i = \begin{cases} 1 & x \le a_i \\ 0 & x > a_i \end{cases} \qquad i = 1, 2 .$$

Model derivation

The Max and Burkhart's taper model was fitted using the NLMIXED procedure in SAS. Once the best combination of random parameters and fixed parameters was identified, the random parameters were substituted by CR and stand density.

When considering crown ratio effects, the combination of b_1 and b_2 as random effects was best. MB taper model with random parameters b_1 and b_2 substituted by crown ratio resulted in the following taper equation:

$$\begin{aligned} y = {} & (b_1 + \lambda_1 CR)(x-1) + (b_2 + \lambda_2 CR)(x^2-1) + \\ & b_3(a_1-x)^2 I_1 + b_4(a_2-x)^2 I_2 \end{aligned} \quad (2)$$

where $CR = CL/H$, CL is tree crown length (m), λ_i is regression coefficients, $i = 1, 2$, All other variables as previously defined.

When considering stand density effects, the combination of b_1 and b_2 as random effects was best. However, MB taper

model with random parameters b_1 and b_2 substituted by basal area (BA) failed to converge, the combined equation system was used as following:

$$\begin{cases} y = (b_1 + u_1)(x-1) + (b_2 + u_2)(x^2 - 1) + \\ b_3(a_1 - x)^2 I_1 + b_4(a_2 - x)^2 I_2 \\ u_1 = \lambda_1 + \lambda_2 BA \\ u_2 = \lambda_3 + \lambda_4 BA \end{cases} \tag{3}$$

where BA is basal area (m^2/ha), u_i is a random parameter, $i = 1, 2$, λ_i is regression coefficients, $i = 1...4$, All other variables are as previously defined.

Parameter estimation

Three equations then were compared: (1) Max and Burkhart taper model (MB), (2) Max and Burkhart taper model with some of the parameters substituted by crown ratio (MBCR), and (3) Max and Burkhart taper model with some of the parameters substituted by stand density (MBD). Mixed-effects modeling procedure was not used for model fitting because an important limitation when implementing this methodology is the necessity of upper stem diameter-height measurements (calibration) for improving diameter prediction. The NLMIXED procedure was only used to find the random parameters. The following two methods were used for model fitting.

(1) The SAS procedure PROC NLIN was used to estimate parameters for MB and MBCR models. Three main iterative options in the SAS NLIN procedure include the Gauss-Newton, Marquardt, and the steepest decent method. The Marquardt method is most useful when the parameter estimates are highly correlated. In addition, parameter estimates can be obtained with the Marquardt method even when the Gauss-Newton method fails to converge. Marquardt method was used in PROC NLIN procedure.

(2) Model 3 was fitted simultaneously using SYSNLIN procedure in SAS. u_1 and u_2 are endogenous variables because they appear on both the left and right hand sides of these equations. To eliminate simultaneous equation bias, predicted rather than observed u_1 and u_2 values were used as regressors in model 3.

Model evaluation

Root mean square error (RMSE) and coefficient of determination (R^2) as described were employed for model evaluation (Sharma and Zhang 2004). These evaluation statistics are defined as:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2}{\text{n - k}}}$$

$$R^2 = 1 - \left[\frac{\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2}{\sum_{i=1}^{n}\left(Y_i - \bar{Y}\right)^2}\right]$$

where Y_i is observed value for the i^{th} observation, $\hat{Y}_i$ is predicted value for the i^{th} observation, $\bar{Y}$ is mean of the Y_i, k is the number of estimated parameters, n is the number of observations in the dataset.

A cross-validation approach was used to evaluate the prediction performance of the taper models. The dataset was randomly divided into two groups. Each group included fifty percent of sample trees. We fit taper equation to one group, and then check the capability of the equation to predict responses for the group omitted. Repeating this for each group, and then we measure the overall capability of each taper equation. RMSE were calculated for all tested observations.

To test whether the overall addition of the CR and stand density significantly influences the taper equation, a nonlinear extra sum of squares method was employed (Neter et al. 1996). This method requires the fitting of full and reduced modes. In this study, the full model represents MBCR or MBD. The reduced model represents MB. The significance of the full and reduced model comparisons are based on an F-test of the form:

$$F = \frac{(SSE_R - SSE_F)/(df_R - df_F)}{SSE_F / df_F}$$

where SSE_R is the error sum of squares of the reduced model, SSE_F is the error sum of squares of the full model, df_R is the degrees of freedom for the reduced model, df_F is the degrees of freedom for the full model. Generally, the F-test was considered significant if the P-value for the test is less than 0.05.

Results and discussion

Parameter estimates and fit statistics for outside and inside bark taper equations with and without CR and BA are displayed in Tables 2 and 3. All parameters in all models were significant (p<0.0001). All models explained more than 98% of the variation of diameter outside and inside bark. MB taper model with addition of the CR and BA variables provided more efficient parameter estimates when compared to MB taper model. The MBCR and MBD taper models provided smaller standard errors than comparable MB model parameters. Inclusion of the CR and BA variables did not show substantial differences for R^2 values. However, the root mean square error (RMSE) for diameter outside and inside bark decreased respectively by 10% and 7% in MBCR taper model and by 12% and 7.2% in MBD taper model.

Table 2. Parameter estimates (standard errors in parentheses) and model fit statistics for outside bark taper equations for dahurian larch sample trees

Parameter	MB	MBD	MBCR
b_1	-5.1875 (0.5033)	-5.0915 (0.3695)	-4.0073 (0.3979)
b_2	2.5742 (0.2746)	2.4857 (0.2020)	1.7834 (0.2230)
b_3	-2.5928 (0.2506)	-2.5612 (0.1821)	-2.5207 (0.1883)
b_4	110.1120 (14.1138)	83.6715 (7.7694)	87.7938 (8.9445)
a_1	0.7671 (0.0234)	0.7540 (0.0198)	0.7544 (0.0209)
a_2	0.0750 (0.0052)	0.0873 (0.0045)	0.0849 (0.0047)
λ_1		-0.8717 (0.0172)	-1.6942 (0.1263)
λ_2		0.0349 (0.0007)	1.1865 (0.0924)
λ_3		0.6260 (0.0130)	
λ_4		-0.0250 (0.0005)	
RMSE	0.8370	0.7380	0.7545
R^2	0.9825	0.9864	0.9858

Note: MB, Max and Burkhart taper model; MBD, Max and Burkhart taper model with some of the parameters substituted by stand density; MBCR, Max and Burkhart taper model with some of the parameters substituted by crown ratio.

Table 3. Parameter estimates (standard errors in parentheses) and model fit statistics for inside bark taper equations for dahurian larch sample trees

Parameter	MB	MBD	MBCR
b_1	-4.3582 (0.3543)	-4.1983 (0.2891)	-3.6262 (0.3126)
b_2	2.1875 (0.1933)	2.0919 (0.1576)	1.6441 (0.1748)
b_3	-2.1360 (0.1752)	-2.0809 (0.1417)	-2.0678 (0.1483)
b_4	108.4064 (17.8813)	93.4330 (12.3125)	87.1387 (10.9517)
a_1	0.7595 (0.0216)	0.7516 (0.0200)	0.7498 (0.0207)
a_2	0.0665 (0.0058)	0.0720 (0.0051)	0.0748 (0.0051)
λ_1		-0.4271 (0.0135)	-0.9949 (0.1029)
λ_2		0.0156 (0.0003)	0.7978 (0.0745)
λ_3		0.3552 (0.0091)	
λ_4		-0.0129 (0.0002)	
RMSE	0.7422	0.6888	0.6902
R^2	0.9843	0.9865	0.9864

MB, MBCR and MBD taper models were further analyzed based on the cross-validation. To evaluate the predictive ability of the models over the entire length of the stem, the independent variable, relative height, was divided into ten sections. RMSE were calculated for each section of diameter outside and inside bark. For comparison, a graphical examination of the performance of the three taper equations was conducted (Fig. 1). For diameter outside bark, MBD model showed good performance for the lower section (relative heights, <30%) and upper section (relative heights, >50%) with the lower RMSE values. MBCR model performed better than the MB model at the lower section (relative heights, <10%), middle section (relative heights, 20%–30%), and upper section (relative heights, >50%). MBD model performed better than the MB model at the lower section (relative heights, <10%) and upper section (relative heights, >40%). MBCR model performed better than the MB model at the lower section (relative heights, <10%) and upper section (relative heights, 40–90%) for inside bark diameter estimates. Overall, the MBCR and MBD taper models showed consistent performance for each section and did not exhibit large variation in any section. The MBCR and MBD models were superior to MB model for most sections in predicting diameter outside and inside bark.

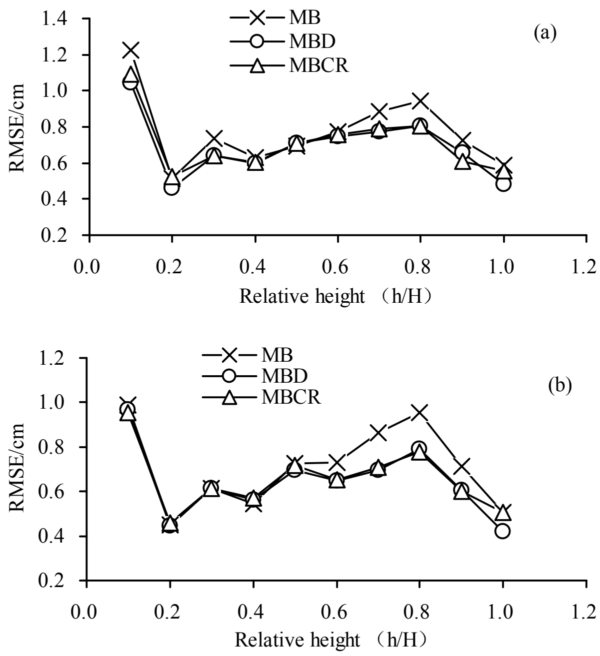

Fig. 1 Root mean square error (RMSE) by relative height classes (h/H) for models MB, MBD, and MBCR. (a) diameter outside bark (b) diameter inside bark

To test whether the overall addition of the CR and BA variables significantly improved the taper equation, a nonlinear extra sum of squares procedure was used. F-tests were conducted between the full model and the reduce model (Table 4). The results indicated that there were significant differences for all the models compared (p<0.0001). Inclusion of the CR and BA variables improved the fit and predictive abilities of MB taper models for Dahurian larch. This agrees with the statement that crown size and stand density influence the changes in taper (Muhairwe et al. 1994; Sharma and Zhang 2004; Jiang et al. 2007).

To further analyze the characteristics of the MBCR model, a series of taper curves were generated for trees having different total height (H), dbh, and CR values (Fig. 2). All parameter values are based on the parameter estimates from Table 3. The characteristics of the simulated trees are: (1) dbh = 12 cm, H =

12 m, CR = 0.25 and 0.65; (2) dbh = 16 cm, H = 16 m, CR = 0.25 and 0.65; (3) dbh = 20 cm, H = 20 m, CR = 0.25 and 0.65. Fig. 2 indicates that trees of the same relative size but with a larger crown ratio, generally have a smaller stem size and thus lower stem volume in the upper bole section due to greater taper in the long crown section. These relationships identified in this study are consistent with results reported by Valenti and Cao (1986), Farrar (1987), and Muhairwe et al. (1994) for some pine species. The relationship is also consistent with results obtained by Jiang et al. (2007) for yellow-poplar in West Virginia.

Table 4. F-tests for the Max and Burkhart model with and without crown ratio and stand basal area

	Full Model		Reduced Model			
models pair	df_F	SSE_F	df_R	SSE_R	*F*-value	*P*-value
Outside bark						
MBD–MB	753	412	755	529	106.9	<0.0001
MBCR-MB	753	429	755	529	87.8	<0.0001
Inside bark						
MBD–MB	753	348	755	416	73.6	<0.0001
MBCR-MB	753	359	755	416	59.8	<0.0001

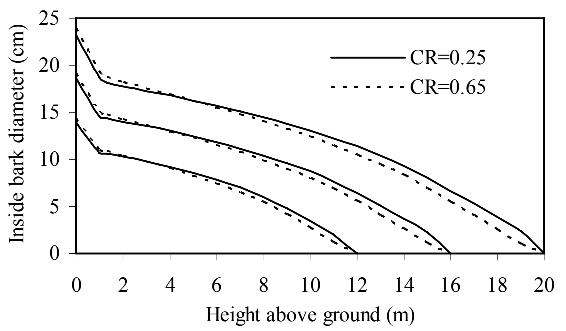

Fig. 2 Tree profiles generated from model MBCR by the different values of dbh (12, 16, and 20 cm), total height (12, 16, and 20 m) and different CR values (0.25 and 0.65)

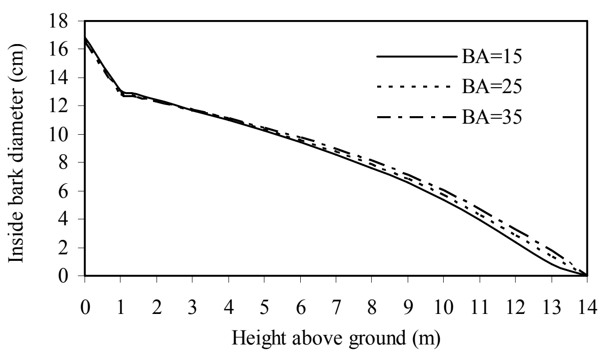

Fig. 3 Tree profiles generated from model MBD by the values of dbh (14 cm), total height (14 m) and different basal area values (15, 25, and 35 m^2/ha)

Taper curves were also generated for a tree with a dbh of 14 cm and a total height of 14 m using MBD taper model at three different basal areas (15, 25, and 35 m^2/ha) (Fig. 3). All parameter values are based on the parameter estimates from Table 3. Fig. 3 shows that trees in lower basal area stand have more taper and lower stem volume in the upper bole section. The relationship was also noted by Sharma and Zhang (2004) for black spruce using stand density (trees per hectare) at the time of plot establishment. However the relative differences in lower stem size were smaller in our study for Dahurian larch than in Sharma and Zhang's (2004) study for black spruce.

It should be noted that correlated error structure in the data was not taken into account in the NLIN and SYSNLIN procedure. Prediction accuracy is little affected by the correlated error structure, even when the correlated errors structure is accounted for in the equation fitting process (Kozak 1997). The autocorrelation are generally ignored and has no use in practical applications.

Conclusions

In this study, the Max and Burkhart's (1976) taper equation was proposed for Dahurian larch in Northeastern China. The equation showed good performance for predicting diameter outside and inside bark in terms of overall fit statistics, sectional performance, and cross-validation. Incorporating crown ratio and basal area into the segmented taper equation improved predictive abilities over the original taper equation. Crown ratio and stand density influence the changes in taper. For a particular height, the trees grown in higher density stands have smaller taper than that grown in lower density stands. Trees of the same relative size but with a larger crown ratio generally have larger taper than that with smaller crown ratio. The additional cost of measuring these variables is not justified in this study.

References

Bi H. 2000. Trigonometric variable-form taper equations for Australian eucalyptus. *For Sci*, **46**: 397–409.

Brooks JR, Jiang L, Clark A. 2007. Compatible stem taper, volume, and weight equations for young longleaf pine plantations in Southwest Georgia. *South J Appl For*, **31**(4): 187–191.

Brooks JR, Jiang L, Ozcelik R. 2008. Compatible stem volume and taper equations for Brutian Pine, Taurus Cedar, and Taurus Fir in Turkey. *For Ecol Manage*, **256**: 147–151.

Fang Z, Borders BE, Bailey RL. 2000. Compatible volume taper models for loblolly and slash pine based on system with segmented-stem form factors. *For Sci*, **46**: 1–12.

Farrar RMJr. 1987. Stem-profile functions for predicting multiple-product volumes in natural longleaf pines. *South J Appl For*, **11**(3): 161–167.

Jiang L, Brooks JR. 2008. Taper, volume, and weight equations for red pine in West Virginia. *North J Appl For*, **25**: 151–153.

Jiang L, Brooks JR, Wang J. 2005. Compatible taper and volume equations for yellow-poplar in West Virginia. *For Ecol Manage*, **213**: 399–409.

Jiang L, Brooks JR, Hobbs GR. 2007. Using crown ratio in yellow-poplar compatible taper and volume equations. *North J Appl For*, **24**: 271−275.

Kozak A. 1988. A variable-exponent taper equation. *Can J For Res*, **18**: 1363−1368.

Kozak A (1997) Effects of multicollinearity and autocorrelation on the variable-exponent taper functions. *Can J For Res*, **27**: 619−629.

Kozak A. 2004. My last words on taper equations. *For Chron*, **80**: 507−515.

Larson PR. 1963. Stem form development of forest trees. *For Sci Monogr,* **5**: 42.

Leites LP, Robinson AP. 2004. Improving taper equations of loblolly pine with crown dimensions in mixed-effects modeling framework. *For Sci*, **50**: 204−212.

Max TA, Burkhart HE. 1976. Segmented polynomial regression applied to taper equations. *For Sci*, **22**: 283−289.

Muhairwe CK. 1994. Tree form and taper variation over time for interior lodgepole pine. *Can J For Res*, **24**: 1904−1913.

Neter J, Kutner MH, Nachtsheim CJ, Wasserman W. 1996. *Applied linear statistical models*. New York: McGraw-Hill, p.1048 .

Sharma M, Oderwald RG. 2001. Dimensionally compatible volume and taper equations. *Can J For Res,* **31**: 797−803.

Sharma M, Zhang SY. 2004. Variable-exponent taper equations for jack pine, black spruce, and balsam fir in eastern Canada. *For Ecol Manage*, **198**: 39−53.

Trincado G, Burkhart HE. 2006. A generalized approach for modeling and localizing stem profile curves. *For Sci*, **52**: 670−682.

Valenti MA, Cao QV. 1986. Use of crown ratio to improve loblolly pine taper equations. *Can J For Res*, **16**: 1141−1145.

Zakrzewski WT, MacFarlane DW. 2006. Regional stem profile model fro cross-border comparisons of harvested red pine (*Pinus resinosa* Ait.) in Ontario and Michigan. *For Sci*, **52**: 468−475.